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# The Propensity to Disruption for Evaluating a Parliament

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**Abstract** The issue of power plays a relevant role in evaluating the representativeness of a Parliament. In this paper a new governability index is introduced, taking inspiration from the propensity to disruption and referring to the power of the parties.

**Keywords** Electoral system, representativeness, governability, simulation
**JEL classification** D72, C70

## 1. Introduction

An electoral system, or voting system, may be viewed as a mechanism for transforming the preferences of a population into a Parliament. It corresponds to the set of rules governing the various steps of the election, from how the voters express their preferences on the parties till how the seats are assigned to the parties. It is natural to pose the question of which system is preferable, but the complexity of the problem and the number of different parameters that may influence the decision may make very hard to give an answer that is suitable for all the possible real-world situations.

Let us consider the following classical example.

**Example 1.** Three parties A, B, C have the following distribution of votes:

| A | B | C |
|---|---|---|
| 49% | 49% | 2% |

The perfect proportional system (PP) allocates $K$ seats as:

| A | B | C |
|---|---|---|
| $\frac{49K}{100}$ | $\frac{49K}{100}$ | $\frac{2K}{100}$ |

When $K$ is not a multiple of 100, it is necessary to round the result. Supposing that the Parliament is made up by 6 seats the ratios in the previous table are 2.94, 2.94, 0.12, respectively, so the following assignments, each corresponding to a different voting system, seem reasonable for representing voters' preferences (Table 1).

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**Table 1.** Seats assignments

| | A | B | C |
|---|---|---|---|
| $h_1$ | 3 | 3 | 0 |
| $h_2$ | 3 | 2 | 1 |
| $h_3$ | 2 | 3 | 1 |

Note that in all the three systems, $h_1$, $h_2$ and $h_3$, at least one pair of parties does not generate a majority, while according to the votes distribution all pairs represent a possible majority.

This simple example shows that it is difficult to take into account all the elements that may influence the quality of an electoral system. In this paper we deal with the role of power, in order to improve the correct evaluation of a voting system. In other words, we want to evaluate an electoral system more on the basis of the power assigned to the parties than of the number of seats they receive. The concept of power may be defined and measured in different ways. Here, we refer mainly to the influence of a party in forming a majority, e.g. the consequence of its behavior in favor or against the approval of a law, and measure it using game theoretical indices of power.

The structure of the paper is as follows: In the next section we recall some basic definitions of game theory; Section 3 presents two of the most largely accepted parameters for evaluating a Parliament, namely the representativeness and the governability; Section 4 is devoted to the introduction of two new indices based on the power assigned to the parties; finally, some remarks conclude.

## 2. Some recalls of game theory

A *cooperative game with transferable utility* or TU-game, is a pair $(N,v)$, where $N=\{1,2,...,n\}$ denotes the finite set of *players* and $v:2^N \to \mathbb{R}$ is the *characteristic function*, with $v(\emptyset)=0$. $v(S)$ is the *worth* of coalition $S \subseteq N$, i.e. what players in $S$ may obtain standing alone.

A game is *simple* when $v:2^N \to \{0,1\}$, *monotonicity* holds, i.e. $S \subseteq T \Rightarrow v(S) \leq v(T)$, and $v(N)=1$. If $v(S)=0$ then $S$ is a *losing* coalition, while if $v(S)=1$ then $S$ is a *winning* coalition.

Given a simple game, $i \in N$ is a *veto* player if $v(S)=0$, for each $S \subseteq N \setminus \{i\}$, i.e. all coalitions not including player $i$ are losing.

A particular class of simple games are the *weighted majority games*. The players are associated to a weight vector $w=(w_1,w_2,\ldots,w_n)$ that leads to the following definition of the characteristic function of the corresponding weighted majority game $(N,w)$:

$$w(S)=\begin{cases} 1 & \text{if } \sum_{i\in S} w_i > q \\ 0 & \text{otherwise} \end{cases},$$

where $q$ is the *quota*. Usually we ask that $q \geq 1/2\sum_{i\in N} w_i$, so that if $S$ is winning $N \setminus S$ is losing.

An *allocation* is a $n$-dimensional vector $(x_i)_{i \in N} \in \mathbb{R}^N$ assigning to player $i \in N$ the amount $x_i$; an allocation $(x_i)_{i \in N}$ is *efficient* if $x(N) = \sum_{i \in N} x_i = v(N)$. A *solution* is a function $\psi$ that assigns an allocation $\psi(v)$ to every TU-game belonging to a given class of games $\mathcal{G}$ with player set $N$.

Two classical solutions for TU-games are the Shapley value (Shapley 1953) and the Owen value (Owen 1977). The Shapley value assigns to each player his average marginal contribution over all the possible permutations of the players:

$$\phi_i(v) = \frac{1}{n!} \sum_{\pi} \left[ v(P(\pi;i) \cup \{i\}) - v(P(\pi;i)) \right],$$

where $\pi$ is a permutation of the players and $P(\pi;i)$ is the set of players that precede player $i$ in the permutation $\pi$. The Shapley value may be rewritten as:

$$\phi_i(v) = \sum_{S \subseteq N \setminus \{i\}} \frac{s!(n-s-1)!}{n!} \left[ v(S \cup \{i\}) - v(S) \right],$$

where $n = |N|$ and $s = |S|$. ($|S|$ denotes the cardinality, i.e. the number of elements, of a set $S$.)

For a simple game the Shapley value is referred to as the Shapley-Shubik index (Shapley and Shubik 1954).

The Owen value is a coalitional version of the Shapley value, i.e. the players are supposed to be structured according to *a priori unions*, i.e. the player set $N$ is partitioned as $K = \{T_1, \ldots, T_k\}$, with $T_i \cap T_j = \emptyset, i \neq j$ and $\bigcup_{i=1,\ldots,k} T_i = N$. The Owen value can be written as:

$$\Omega_i(K) = \sum_{\substack{H \subseteq K \\ T_j \notin H}} \sum_{\substack{S \subseteq T_j \\ i \notin S}} \frac{h!(k-h-1)!s!(t_j-s-1)!}{k!t_j!} \left[ v(H \cup S \cup \{i\}) - v(H \cup S) \right],$$

where $h = |H|, k = |K|$ and $t_j = |T_j|$.

## 3. How to evaluate a parliament?

According to the current literature, the choice of the "best" parliament may be affected by a lot of facets of the political process, but two of them may be considered more relevant than the others: *representativeness* that depends on the efficiency of the system in representing electors' preferences and *governability* that measures the effect on the efficiency of the resulting government. The resulting government depends also on the choices of the parties, but nevertheless we may fix a rule for forming a majority and we may apply this rule to all the possible parliaments associated to several voting systems; we refer to Rusinowska et al. (2004) for an analysis of possible majority formation rules. We address the reader to Fragnelli et al. (2005) for a list of possible parameters for evaluating a parliament and related references. Representativeness and governability can be measured by two indices, $r$ and $g$ respectively, normalized in the interval $[0,1]$, that can be defined in several ways.

As pointed out in Fragnelli et al. (2005), when we restrict the analysis to a single election the comparison among several electoral systems using the two parameters $r$ and $g$, we may have the following situations:

(i) A system may have worse values for both indices w.r.t. another system, i.e. the first system is *dominated*, consequently it may be excluded.

(ii) A system may have better values for both indices w.r.t. all the other systems, i.e. the first system is *dominant*, consequently it is the best system.

(iii) A system may be neither dominated or dominant, i.e. if an index has a better value the other index has a worse value w.r.t. another system, consequently all these systems are *Pareto optimal*.

Of course, not always a dominant system exists, so that the choice among the Pareto optimal systems requires other tools, that are beyond the aim of this paper, and for which we refer to Fragnelli et al. (2005).

The measure of the representativeness of a voting system may be obtained on the basis of the distribution of votes and of seats. In Fragnelli et al. (2005), the representativeness is measured on the basis of the difference between *the votes casted in a nation-wide proportional district and the seats assigned by a given electoral system h*:

$$r_h = 1 - \frac{\sum_{i \in N} |S_i^h - S_i^{PP}|}{\sum_{i \in N} |S_i^u - S_i^{PP}|},$$

where $N$ is the set of parties, $S_i^h$ is the number of seats of party $i$ with system $h$, $S_i^{PP}$ is the number of seats of party $i$ with the perfect proportional system and $S_i^u$ is equal to the total number of seats for the relative majority party under system $h$, i.e. applying the option *the winner takes all*, and is equal to 0 otherwise.

Computing this index for the three systems $h_1$, $h_2$ and $h_3$ in Example 1, we obtain 0.961, 0.693 and 0.693, respectively.

Again, in Fragnelli et al. (2005) the governability is represented by the following index:

$$g_h = \frac{1}{m_h + 1} + \frac{1}{m_h(m_h + 1)} \frac{f_h - \frac{T}{2}}{\frac{T}{2}},$$

where $m_h$ is the number of parties of the governing coalition under system $h$ that may destroy the majority if they withdraw, $f_h$ is the number of seats of the majority under system $h$ and $T$ is the total number of seats in the Parliament.

Supposing that the governing coalition corresponds to the minimal winning coalition that includes the relative majority party, we may compute this index for the three systems $h_1$, $h_2$ and $h_3$ in Example 1, obtaining 0.500 for $h_1$ and 0.444 for $h_2$ and $h_3$ when the majority is formed by parties A and B or 0.389 if the majority includes party C.

Now we introduce another system, $h_4$, that assigns 2 seats to each party; for this system the representativeness index is 0.386 (supposing that under system $u$ all seats go to party A or B) or 0.680 (supposing that under system $u$ all seats go to party C) and the governability index is 0.389. So, we may conclude that the system $h_1$ is "the best" as it dominates the other three systems.

Computing the weighted majority games associated to the distribution of votes, $v$, using the percentages as weights and to the distribution of seats according to the voting systems $h_1$, $h_2$, $h_3$ and $h_4$, with the number of seats in the role of weights and using the simple quota $q = 1/2 \sum_{i \in N} w_i$, we obtain values in Table 2:

**Table 2.** Weighted majority games (see Example 1)

| Voting system | Coalitions | | | | | | |
|---|---|---|---|---|---|---|---|
| | A | B | C | AB | AC | BC | ABC |
| $v$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| $h_1$ | 0 | 0 | 0 | 1 | 0 | 0 | 1 |
| $h_2$ | 0 | 0 | 0 | 1 | 1 | 0 | 1 |
| $h_3$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 |
| $h_4$ | 0 | 0 | 0 | 1 | 1 | 1 | 1 |

It is easy to check that the games associated to systems $h_1$, $h_2$ and $h_3$ have at least one veto player, while the one associated to system $h_4$ has no veto players, as the game associated to $v$.

## 4. The role of power

In this section we switch our attention to the power of the parties and look for the relationship among the power they have according to the distribution of votes and according to the distribution of seat. We use as power measure of the parties the Shapley-Shubik index of the weighted majority game associated to the distribution of votes and to the distributions of seats.

We want to remark that other indices could be used but the aim of this paper is methodological, i.e. it proposes to use the power of the parties in the evaluation of a Parliament, more than technical, i.e. computing the "right value" of a Parliament. We just mention as possible indices those by Banzhaf (1965), Deegan and Packel (1978), and Holler (1982). The motivation behind the choice of the Shapley-Shubik index is that for this index there exists the coalitional version, which will enable us to emphasize the role of the majority coalition (see Subsection 4.2).

### 4.1 Power and representativeness

We proposes a representativeness index based on power starting from the idea of measuring the distance of the distribution of power on the votes and on the seats, i.e. $\sum_{i \in N} |\phi_i^v - \phi_i^h|$. This distance is zero when the power of each party is identical in the two distributions. Consequently, a possible representativeness index is given by $1 - \sum_{i \in N} |\phi_i^v - \phi_i^h|$.

**Remark 1.** *The representativeness is maximal, i.e. equal to 1, when the power of each party is identical in the two distributions. A particular case happens when the percentages of votes coincide with the percentages of seats, e.g. in the pure proportional system, according to standard literature.*

It is possible that the index assumes a negative value, as in the following example.

**Example 2.** Consider a relative majority system with two parties A and B and three districts $D_1$, $D_2$ and $D_3$, each one electing a single member. The results of the elections and the assignment of the seats are in Table 3:

**Table 3.** A simple voting system

| Parties | $D_1$ | $D_2$ | $D_3$ | % of votes | # of seats |
|---|---|---|---|---|---|
| A | 16 | 6 | 8 | 60 | 1 |
| B | 2 | 8 | 10 | 40 | 2 |

The weighted majority game on votes, $w^v$, and on seats, $w^h$, are $w^v(\text{A}) = 1, w^v(\text{B}) = 0, w^v(\text{AB}) = 1$ and $w^h(\text{A}) = 0, w^h(\text{B}) = 1, w^h(\text{AB}) = 1$, respectively. In this case $\phi^v = (1,0)$ and $\phi^h = (0,1)$, so the index is $1-2 = -1$.

To avoid this situation we can normalize the index in the interval $[0,1]$, simply dividing $\sum_{i \in N} |\phi_i^v - \phi_i^h|$ by 2, as in the worst case the two distributions of power may assign complementary values.[1] So, we have:

$$r^\Omega = 1 - \frac{\sum_{i \in N} |\phi_i^v - \phi_i^h|}{2}$$

**Example 3.** Referring to Example 1 we have:

**Table 4.** The new representativeness index (see Example 1)

| Voting system | $v$ | $h_1$ | $h_2$ | $h_3$ | $h_4$ |
|---|---|---|---|---|---|
| $\phi$ | $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ | $\left(\frac{1}{2},\frac{1}{2},0\right)$ | $\left(\frac{4}{6},\frac{1}{6},\frac{1}{6}\right)$ | $\left(\frac{1}{6},\frac{4}{6},\frac{1}{6}\right)$ | $\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$ |
| $r^\Omega$ | – | $\frac{2}{3}$ | $\frac{2}{3}$ | $\frac{2}{3}$ | 1 |

So, the system $h_4$ that had the worst representativeness according to the previous index obtains the maximal performance, with the same power distribution of the votes.

[1] Two vectors are complementary when each non-zero component in a vector corresponds to a zero component in the other vector.

This is not the first case in which power is used for evaluating representativeness. Gambarelli and Biella (1992) propose to measure the distance of two distributions referring to the percentages of distribution of voters, $v$, to the assignment of seats according to an electoral system $h$, $s^h$, and to the power of the parties related to the votes and to the seats, $\varphi^v$ and $\varphi^h$ respectively. The resulting distance $\Delta$ is:

$$\Delta = \max_{i \in N} \left\{ |v_i - s_i^h|, |\varphi_i^v - \varphi_i^h| \right\}$$

Referring to the Shapley value as power index we get that $\Delta = 0.313$ for $h_4$ and $\Delta = 0.333$ for the other systems. $1 - \Delta$ can be used as a representativeness index that gives the value 0.687 for $h_4$ and 0.667 for the other systems.

### 4.2 Power and governability

Our motivation for this subsection arises from the following question: Why not to use power indices also for governability?

We were inspired by the propensity to disrupt (see Gately 1974), that measures how much a proposed allocation $x$ is satisfactory for player $i$. Suppose that the grand coalition $N$ forms and the players agree on an allocation $x = (x_1, x_2, \dots, x_n)$; if player $i \in N$ recedes from $N$, he receives $v(i)$ instead of $x_i$ and the other players receive globally $v(N \setminus \{i\})$ instead of $x(N \setminus \{i\})$. The larger is $x_i - v(i)$ the less is the propensity to disrupt for player $i$ and the larger is $x(N \setminus \{i\}) - v(N \setminus \{i\})$ the less the other players accept that player $i$ recedes. Gately defined the *propensity to disrupt* as the ratio $\big[x(N \setminus \{i\}) - v(N \setminus \{i\})\big] / [x_i - v(i)]$.

**Remark 2.** *Note that* $\sum_{i \in S} (x_i - v(i))$ *and* $\big(x(S \setminus \{i\}) - v(S \setminus \{i\})\big)$ *may be used as indices of the stability of a parliamentary coalition* $S \subseteq N$.

When $S$ represents a majority coalition, it is possible to emphasize its power using the Owen value, $\Omega$, instead of the Shapley-Shubik index, that assigns null power to each player not in the majority.

We propose to take into account the quantity

$$\Omega_i(S) - \Omega_i(S \setminus \{i\}), \quad i \in S,$$

where $\Omega(S)$ is the Owen value when the coalition structure $K_S$ considers the coalition $S$ and all the other players as singletons, i.e. $K_S = \{S, \{i_1\}, \dots, \{i_{n-s}\}\}$ and $K_{S \setminus \{i\}} = \{S \setminus \{i\}, \{i\}, \{i_1\}, \dots, \{i_{n-s}\}\}$, where $s = |S|$. If we consider the difference of power of party $i$ when it is in a majority coalition $S$ and when it leaves the majority $\Omega_i(S) - \Omega_i(S \setminus \{i\})$, we have a measure of the propensity of party $i$ to leave the majority. Consequently, the higher is the propensity of the parties to stay in the majority, the higher is the stability of the majority.

Summing up on all the players in $S$, we get the following governability index $\sum_{i \in S} \big[\Omega_i(S) - \Omega_i(S \setminus \{i\})\big] = 1 - \sum_{i \in S} \Omega_i(S \setminus \{i\})$, where we used that if $S$ is a majority then $\sum_{i \in S} \Omega_i(S) = 1$.

**Remark 3.** *According to the standard literature, the governability is maximal, i.e. equal to 1, when $S$ is such that each subcoalition $S \setminus \{i\}$ is winning, i.e. the majority is not affected whichever party leaves it. Note that in this case for each party $i \in S$, $\Omega_i(S \setminus \{i\}) = 0$.*

Again, it is possible that the index assumes a negative value, as in Example 4.

**Example 4.** Consider a weighted majority game $(N, w)$ in which the unique winning coalition is the grand coalition. In this case $\Omega_i(N) = 1/n$, $i \in N$; if party $k$ leaves the majority its power becomes $\Omega_k(N \setminus \{k\}) = 1/2$ and the power of the other $n-1$ parties becomes $\Omega_i(N \setminus \{k\}) = 1/2(n-1)$. So, the index is $1 - \sum_{i \in N} \Omega_i(N \setminus \{i\}) = 1 - \sum_{i \in N} 1/2$ that is negative when $n \geq 3$.

To avoid this situation we can normalize the index in the interval $[0,1]$, simply dividing $\sum_{i \in N} \Omega_i(N \setminus \{i\})$ by $n$, as $0 \leq \Omega_i(N \setminus \{i\} \leq 1$ for each $i \in N$. Hence:

$$g^{\Omega} = 1 - \frac{\sum_{i \in S} \Omega_i(S \setminus \{i\})}{n}$$

**Example 5.** Referring to Example 1 we have the following values $g^{\Omega}_{h_1} = 0.667$, $g^{\Omega}_{h_2} = g^{\Omega}_{h_3} = 0.722$, $g^{\Omega}_{h_4} = 0.778$. Again, the system $h_4$ that had the worst governability according to the previous index obtains the best value and taking into account both indices $r^{\Omega}$ and $g^{\Omega}$ the system $h_4$ results to be dominant.

The following example taken from Fragnelli et. al (2005) is used for a comparison of the indices based on power with the indices used in Fragnelli et. al (2005), referring to a situation that is more realistic than the one considered in the previous examples.

**Table 5.** Seats assignment after simulation of different voting systems

| Voting system[2] | $P_1$ | $P_2$ | $P_3$ | $P_4$ | $P_5$ | $P_6$ | $P_7$ | $P_8$ | $P_9$ | $P_{10}$ | $P_{11}$ | $P_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $PP(=v)$ | 18 | 22 | 5 | 4 | 0 | **5** | 0 | **1** | 0 | **17** | **28** | 0 |
| $P-4$ | 19 | 22 | 5 | **4** | 0 | **5** | 0 | 0 | 0 | **17** | **28** | 0 |
| $P(20)$ | 14 | 16 | 3 | 3 | 0 | 3 | 0 | 1 | 0 | **12** | **48** | 0 |
| $M$ | 14 | **36** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **50** | 0 |
| $2R$ | 14 | **36** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **50** | 0 |
| $C$ | **14** | **36** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | **4** | 46 | 0 |
| $B$ | **9** | **44** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 15 | 32 | 0 |
| $A$ | **18** | **37** | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 18 | 27 | 0 |
| $I-25$ | 15 | 33 | 1 | **1** | 0 | **1** | 0 | 0 | 0 | **4** | **45** | 0 |
| $I-75$ | 17 | 25 | 4 | 3 | 0 | **4** | 0 | **1** | 0 | **13** | **33** | 0 |

| | | | | | |
|---|---|---|---|---|---|
| $PP$ | Pure proportionality | $M$ | Relative majority | $B$ | Borda count |
| $P-n$ | Threshold proportionality | $2R$ | Two-round runoff | $A$ | Approval voting |
| $P(n)$ | Prized proportionality | $C$ | Condorcet method | $I-n$ | Mixed-member |

[2] For a description of the voting systems we refer to Fragnelli and Ortona (2006).

**Example 6.** Consider an electoral situation that involves 12 parties $P_1, P_2, \ldots, P_{12}$, ordered on a left-right axis. Using the simulation program ALEX, developed at Department POLIS of University of Eastern Piedmont (see Bissey et al. 2004), we may compare the resulting Parliament according to ten different voting systems, obtaining Table 5, where the bold numbers identify the parties forming the majority in each system, corresponding to the minimal winning coalition including the relative majority party and the closest parties according to the left-right ordering.

We supposed a unique 100-seat constituency for the pure proportional and threshold proportional systems, a unique $(100-n)$-seat constituency for the prized proportional system, a $n$-seat plus $100-n$ one-seat constituencies for the mixed-member system and 100 one-seat constituencies for all the other systems. Computing the indices we obtain values in Table 6:

**Table 6.** Comparison of old and new representativeness and governability indices, referring to the distribution of seats in Table 5

| Voting system | $r$ | $g$ | $r^\Omega$ | $g^\Omega$ |
|---|---|---|---|---|
| $PP$ | 1.000 | 0.201 | 1.000 | 0.944 |
| $P-4$ | 0.986 | 0.204 | 0.958 | 0.932 |
| $P(20)$ | 0.722 | 0.367 | 0.621 | 0.937 |
| $M$ | 0.500 | 0.453 | 0.676 | 0.931 |
| $2R$ | 0.500 | 0.453 | 0.676 | 0.931 |
| $C$ | 0.556 | 0.257 | 0.801 | 0.931 |
| $B$ | 0.667 | 0.343 | 0.651 | 0.944 |
| $A$ | 0.795 | 0.350 | 0.651 | 0.944 |
| $I-25$ | 0.611 | 0.201 | 0.879 | 0.936 |
| $I-75$ | 0.889 | 0.201 | 0.915 | 0.944 |

We may refer to a graphical representation for the two pairs of indices (using different scales for the governability), see Figure 1.

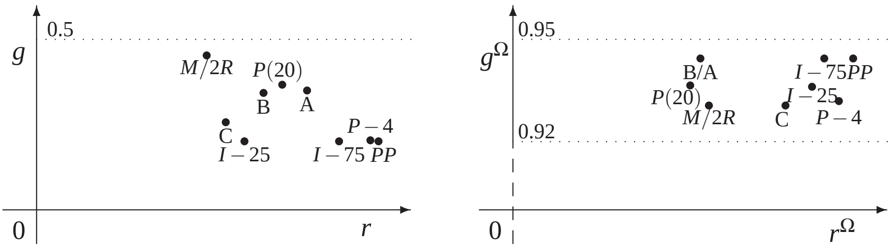


**Figure 1.** Comparison of the electoral systems in Table 5 using old and new representativeness and governability indices

In this case the indices $r$ and $g$ produce six Pareto optimal systems, namely *M*, *2R*, $P(20)$, *A*, $P-4$ and *PP* while the indices $r^{\Omega}$ and $g^{\Omega}$ identify *PP* as a dominant system.

## 5. Concluding remarks and further research

In this paper we introduced two new indices for representativeness and governability, both inspired by the idea of taking into account the issue of power distribution on the parties of a Parliament. The main feature of this approach is that the relevance of a party is related not directly to the number of seats assigned to it by the voting system, but to the power originating by the distribution of seats. Referring again to the example in the Introduction, we want to stress that the system $h_4$ is apparently the less representative but on the other hand is the one that better figures out the effectiveness of parties in forming a majority. Referring to the governability index $g^{\Omega}$, it is possible to follow different hypotheses on the behavior of the parties. For instance, we may suppose a different coalition structure in which there are just two *a priori unions*, one including the parties in the majority and the other one the remaining parties; after that a party leaves the majority it joins to the other union. This leads to a different governability index $g^{\Omega^*} = 1 - \sum_{i \in S} \Omega_i(S \setminus \{i\}, (N \setminus S) \cup \{i\})/n$. This index may be suitable for bipolar situations in which two large parties attract the remaining small parties, that have to decide which of the two large parties join with. Further indices may be defined according to other possible choices of the parties in forming coalitions.

Possible developments of the ideas in this paper are towards a new index that better reflects the idea of Gately, taking into account more explicitly the variations of "payoff" both of each party and of the remaining ones. This point poses at least two challenges: firstly, the difference at denominator, $x_i - v(i)$, is often zero so, according to Gately (1974) the propensity to disrupt goes to $+\infty$ and secondly, in a weighted majority game it is frequent that $v(i)$ is equal to 0 and $v(N \setminus \{i\})$ is equal to 1, so, taking into account that $x(N \setminus \{i\}) = 1 - x(i)$, the propensity index is equal to $-1$ for all the parties, so its utility is limited. Consequently, it is necessary to carefully evaluate how to apply it.

A second possible research field is to analyze the dispersion index by Gini (1914) referred to the power as a possible representativeness index.

A third possibility is offered by an axiomatic characterization of the different indices. This could help in better understanding the underlying features of each index.

Finally, the comparison of different voting systems using the governability index requires fixing a mechanism for the formation of a majority. In fact the governability of a Parliament depends on several aspects, some of them related to the voting system, e.g. the number of parties forming the majority, and some others independent from it, e.g. the behavior of the members of the Parliament. In this framework, the simulative approach could provide useful data, comparing the behavior of the various indices using different voting systems and majority formation processes (see Carlsson et al. 1992; Eklund et al. 2008; and Rusinowska et al. 2004).

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## References

Banzhaf, J. F. (1965). Weighted Voting Doesn't Work: A Mathematical Analysis. *Rutgers Law Review*, 19, 317–343.

Bissey, M. E., Carini, M. and Ortona, G. (2004). ALEX3: A Simulation Program to Compare Electoral Systems. *Journal of Artificial Societies and Social Simulation*, 7(3). [Online version at http://jasss.soc.surrey.ac.uk/7/3/3.html]

Carlsson, C., Ehrenberg, D., Eklund, P., Fedrizzi, M., Gustafsson, P., Merkuryeva, G., Riissanen, T. and Ventre, A. (1992). Consensus in Distributed Soft Environments. *European Journal of Operational Research*, 61, 165–185.

Deegan, J. and Packel, E. W. (1978). A New Index of Power for Simple *n*-Person Games. *International Journal of Game Theory*, 7, 113–123.

Eklund, P., Rusinowska, A. and de Swart, H. (2008). A Consensus Model of Political Decision-Making. *Annals of Operations Research*, 158, 5–20.

Fragnelli, V., Monella, G. and Ortona, G. (2005). A Simulative Approach for Evaluating Electoral Systems. *Homo Oeconomicus*, 22, 524–549.

Fragnelli, V. and Ortona, G. (2006). Comparison of Electoral Systems: Simulative and Game Theoretic Approaches. In Simeone, B. and Pukelsheim, F. (eds.), *Mathematics and Democracy*. Berlin, Springer, 65–81.

Gambarelli, G. and Biella, R. (1992). Sistemi elettorali. *Il Politico*, 164, 557–588.

Gately, D. (1974). Sharing the Gains from Regional Cooperation: A Game Theoretic Application to Planning Investment in Electric Power. *International Economic Review*, 15, 195–208.

Gini, C. (1914). Sulla misura della concentrazione e della variabilità dei caratteri. *Atti del Regio Istituto Veneto di Scienze, Lettere ed Arti*, 73, 1203–1248.

Holler, M. J. (1982). Forming Coalitions and Measuring Voting Power. *Political Studies*, 30, 262–271.

Owen, G. (1977). Values of Games with a Priori Unions. *Lecture Notes in Economic and Mathematical Systems*, 141, 76–88.

Rusinowska, A., de Swart, H. and van der Rijt, J. W. (2004). A New Model of Coalition Formation. *Social Choice and Welfare*, 5, 129–154.

Shapley, L. S. (1953). A Value for *n*-Person Games. In Kuhn, H. W. and Tucker, H. W. (eds.), *Contributions to the Theory of Games, vol. II*. Princeton, Princeton University Press, 307–317.

Shapley, L. S. and Shubik, M. (1954). A Method for Evaluating the Distributions of Power in a Committee System. *American Political Science Review*, 48, 787–792.