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Game-Theoretic Modeling of Electricity Markets in Central Europe

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Abstract The paper deals with the methodology of computer modeling and simulation of complex markets with electricity and related products. The methodology is presented using a particular configuration of Central European markets with decentralized trading and international electricity transfers. The modeling approach is based on pure computer numerical solution in discrete state space determined by problems on which the modeled players are expected to decide—price offered for electricity supplied to various markets, breakdown of total power generation into individual commodities (yearly band, monthly band, spinning reserve) and setting bids in auctions for cross-border profiles. Similar approach to decision-making is adopted on the buyer's side. Buyers are expected to strive to contract power supplies in the way that is most advantageous for them. The generated state space is then analyzed using concepts of mathematical game theory. In this way, we obtain a prediction of probable decisions of modeled players in their market competition. Finally, we present a simplified power system forecast for Central Europe for year 2009.

Keywords Prediction model, algorithmic game theory, modeling, electricity markets
JEL classification C51, C53, C63, C72

1. Introduction

The paper deals with a methodology of computer modeling of complex decision processes related to commodity markets. The whole topic is presented using a case study of electricity markets within the Central Europe. The case study presents a decision situation of many strategic players (producers and consumers), many commodities (a commodity is an electricity supply following some standardized conditions, e.g. time period and time-variant shape of the supply) and many markets (placed in independent transmission systems/countries). Every market is understood to be a point of trading among the local consumer and those producers who are technically capable to supply the national network of the consumer. The players/producers can supply also the markets which are geographically distant from their production plants. In such a case, they

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have to succeed in an auction for network transmission capacities (international electricity network). The mechanism of competing for the transmission capacities is a sort of a multi-object auction and will be referred as the Flow-Based method (FB auction). The export and import can influence the market price in each modeled commodity.

One year is the length of the studied time period. The producers and consumers usually contract the supply for the next coming year a long time in advance. We differentiate between the year supply contracts (YB, year base load commodity) and twelve month supply contracts (MB, month base load commodities). We model a particular region of Central Europe (MCE, Model of Central Europe). For this reason, the case-study (based on the MCE model) contains a true information about the existing producers, the power networks and consumption during the year.

The modeled power plants have their true technical parameters and time variant disponibility. There are also other variable factors affecting the behavior of players like state of the international transmission system which limits the possible supply between the particular countries. By forecasting of the next year market behavior, we mean the forecast of all prices of all studied commodities, description of the signed contracts, prices of the international transmission capacities and many other statistics, e.g. demand of coal and gas.

We propose a solution in form of a computer model able to predict rational behavior of players in given conditions. The proposed methodology is based on analysis of strategic behavior of market players and on a particular concept of game equilibria. The model design comes from a classical concept of game theory—identification of players, definition of their strategy sets, definition of their utility functions and equilibrium determination. We accept the determined equilibrium point as a prediction of probable behavior of the modeled players in reality. All the contracts are signed de facto in a single moment and hence we may model the whole problem as a large and normal-form strategic game (Myerson 2004).

This large normal-form game (given by its players, strategies, utility functions and the equilibrium concept) is too complex to be described analytically (in form of mathematical equations) like in the classical Cournot and Bertrand oligopoly models (Bierman and Fernandez 1998). For this reason, we discretize the domain of the modeled problem into discrete strategy sets S_i of players $i \in \{1,2,\ldots,N\}$. The final equilibrium is determined through a numerical computation within the discrete space of profiles $S = S_1 \times S_2 \times \ldots \times S_N$.

The correlated equilibrium (CE) proposed by Aumann (1974) and later more developed by Papadimitriou (2005) was chosen as a basic equilibrium concept in this prediction model. CE is a well know game theoretic concept extending the classical Nash equilibrium (Nash 1951) with a special synchronization device helping the players to make their decision. A rational player then agrees that incoming event (signal) recommends him the best strategy to choose. This is an opposite to the Nash equilibrium (NE), which assumes no communication platform between players and their surrounding environment. The players then prefer to make careful actions, often leading to lower common social outcome and misunderstandings. Following our experience and results, we do believe that a rational player in market competition (where the rationality is a common knowledge) behaves in the manner of correlated equilibrium. More reasoning for the use of the correlated equilibria has been done in Samuelson (2004). We use a well known algorithm of finding CE based on linear programming (Viguier et al. 2006). The algorithm determines an unique CE where the total outcome is maximized. During our implementation work, the basic algorithm was improved to be more efficient (see Hrubý 2008; Hrubý and Čambala 2008).

1.1 The issue of modeling the electricity markets

The Model of Central Europe (MCE) models a non-trivial multi-player strategic game configuration with structured decision-making being made with a multi-commodity at the market spread among more countries. The MCE is designed for the Central European region consisting of Germany, Poland, Slovakia, Czech Republic, Austria, Hungary and Western Ukraine. However, approaches described in this paper can be applied elsewhere.

Cross-border trading with electricity in Europe is constrained by the topology of the interconnected power system and by capacities of cross-border transmission lines. These constraints restrict the possibility to export electricity for some power producers; at the same time, they effectively restrict some buyers in their rights to buy electricity from non-domestic power sources. International trading thus may affect the price in individual countries. Currently, the allocation of transmission capacities is being carried out at coordinated auctions. In the long-term planning, the allocation of transport capacities is expected to be based on the Flow-Based Method (Glavitsh et al. 2004). So far, however, operators of Central European networks have not reached an agreement regarding the introduction of this method for the year 2009. The current development indicates that either this date will be shifted to 2010 or that the final Flow-Based Method algorithm will substantially change. The MCE model assumes, however, that the Flow-Based Method will be introduced earlier or later.

In our forecasting practice, the MCE model is integrated to a trio of models covering the whole topic of electro-energy industry in Czech Republic and the neighborhood. There are the Model of Central Europe (MCE), Model of long-period contracts in Czech republic (MDK) and the Model of day-ahead market (HM). Each of the models has its special meaning in the forecasting. MCE is of a wide geographical domain and hence it has to reduce its granularity of the modeled detail (mainly the number of strategic players). Its main mission is to estimate probable international market tendencies in the region—it responds the resulting transmission fees arising from the auctions for the international power lines and available transmission capacities. MCE has no ambition to model the studied territory precisely. This is not even possible. The further models (MDK, HM) take over the results from MCE and develop some further forecasting details about the region, contracts, prices and international transfers. As a starting point, MCE is of key importance in this respect.

We should also mention our particular motivation and background for this modeling and forecasting. Under the conditions of the Czech Republic, the MCE model, as a prediction tool, is practically used for the development of long-term balances of electric power production and consumption in the Czech Republic. The development of long-term balances is done in cooperation with Energy Market Operator (EMO) who is by law responsible for these tasks. Activities of EMO also include an organization of day-ahead electricity market. The most important challenge of the Czech national power system is limited capacities of cross-border exchanges, replacement and modernization of the power generation base, securing supplies of primary fuels and meeting environmental targets in context of the EU emission allowances legislation.

2. Mathematical game modeling

A game Γ in strategic form of N players is defined as

$$\Gamma = (Q; S_1, S_2, \ldots, S_N; U_1, U_2, \ldots, U_N; C),$$

where:

(i) $Q = \{1, 2, \ldots, N\}$ is a (finite) set of the players.

(ii) $S_i, i \in Q$ are finite sets of (pure) strategies of players i. Product of strategy sets makes the *game set of profiles* $S = S_1 \times S_2 \times ... \times S_N$. Let $s = (s_1, s_2, \ldots, s_N)$ denote a particular *strategic profile* $s \in S$. Let S_{-i} denote similarly a subspace of S without S_i. S_{-i} notation will be frequently used to express a context of the $i-$th player's decision situation. Finally, s_{-i} will denote a member of S_{-i}.

(iii) $U_i : S \to \mathbb{R}, i \in Q$ are utility functions assigning a payoff to each player i in each profile $s \in S$. In the market games, the payoff means the financial profit of the player in the particular profile $s \in S$. From the computer science of view, the utility functions U_i are usually implemented as N-dimensional arrays indexed by strategy profiles $s \in S$.

(iv) C is a global context of the game, i.e. set of all information generally available to players (C is common knowledge to players).

The strategic profile $s^* \in S$ consisting of the actions $(s_i^*)_{i \in Q}$ made by individual players will be referred as a *game solution*. Players want to choose the best response on their opponent's possible action. The *equilibrium* is the mutually best response, which is formally defined in every book of game theory (Myerson 2004; Bierman and Fernandez 1998; Osborne and Rubinstein 1994). The literature on game theory introduces various forms of the equilibria concepts. We have implemented the correlated equilibrium (Aumann 1974; Papadimitriou 2005).

In market games, competitive situations are subject of modeling, in order to be able to better *understand the behavior of the players* in the real life or to be able to *predict the behavior* of real players (producers, traders and consumers). Theoretical literature on gaming shows sometimes a certain measure of skepticism about whether the game theory can be successfully used for the prediction of future (a very interesting experiment is described in Green 2002); in other cases, the usefulness of the game theory for predictions is defended (Erev et al. 2002). The game theory is undoubtedly a relatively successful and useable method. A number of papers (Kwang-Ho and Baldrick 2003;

Krause et al. 2004; Gountis and Bakirtzis 2004) prove that, using analytic models developing certain game-theoretical principles. As a opposite to these rather theoretical papers, our particular paper (and the MCE model behind) concentrates to a practical computer implementation of the theory of game modeling and decision making.

2.1 Designing a game-theoretic numerical model

When modeling the given strategic situation in form of a game $\Gamma = (Q;S;U;C)$ we split the modeling task and the whole algorithmization to basic two levels (will be refereed as the game level and the internal model level). This approach was published in Hrubý and Toufar (2006) and could be also found in Viguier et al. (2006).

(i) Level of the whole game. We model a game Γ given by its set of profiles S and utility functions $\{U_i\}_{i\in Q}$. By the game-theoretical analysis, we want to determine its probable equilibrium point (or points). The analytical approaches (e.g. equilibrium determination, analysis of strategy dominance) at the game level are well described in the literature. Their efficient algorithmic implementation is a subject of research in computer science (Viguier et al. 2006; Kwang-Ho and Baldrick 2003; Nisan et al. 2007). This paper builds its computing technology to the algorithms described in Hrubý (2008). It is highly recommended to readers to get familiarized with that paper.

(ii) Level of a strategic profile $s \in S$ of a game Γ. Let us define a computer procedure (also called the *internal model* in this paper or an ”oracle” in certain game-theoretical literature) *cellModel* (s,C), that computes for each profile $s \in S$ (and a set of global parameters C) the overall process of planning, trading and managing of the players in the given profiles. The procedure terminates with related utilities $U_i(s)$ of all the players $i \in Q$.

We do not assume that it is possible to formulate analytically the utility functions $U_i : S \to \mathbb{R}$ at the game level. It is too complicated from the point of view of all technical aspects of the player's planning, process of trading and optimization of the production. This all should be included in the utility function. For this reason, we prefer to discretize the domain of the strategic problem and to evaluate sequentially all U_i for all $s \in S$. As a result, we obtain a memory record, an N-dimensional matrix U indexed by the strategic profiles s. Thus, $U_i(s)$ denotes an already enumerated payoff of the player i in the profile s; and $U(s)$ denotes an N-dimensional vector of payoffs of players $1,2,\dots,N$ in the profile s.

From the modeling and software-engineering point of view, this approach is useful to separate the general game-theoretical principle (a software library) and the particular application part (*cellModel*).

The general game-theoretical principle was described in Hrubý (2008). This particular paper develops in detail the mentioned *internal model*, here described in Section 3.

2.2 Design of the game space of profiles

Under real conditions, we often have to make decisions regarding more problems concurrently or, eventually, to adopt decisions in the context of other decisions. The condition of another decision problem increases the dimensionality of the strategy.

In the Cournot or Bertrand oligopoly models, the strategy is regarded as price or quantity. Let us put these dimensions together in the strategic decision making. The strategy $s_i \in S_i$ of a player i is then a two-dimensional vector $s_i = (price, amount)$, $S_i = Prices \times Amounts_i$, where *Prices* is a list of possible prices and $Amounts_i$ is a set of technically possible volumes that can be produced by player i. Modeling of multi-dimensional decisions has already been studied in research literature, usually within the framework of model-based predictions (Hrubý 2007). By choosing a particular strategy $s_i \in S_i$, the player i makes two decisions—he decides the amount to offer and its price.

Let us call them the *multi-dimensional strategies*. If the player i is modeled in such a manner that he has a set of D_i elementary decision-making problems (regarding his productions, prices, markets, etc.), where each elementary sub-problem $d^i_j \in D_i$ belongs to the finite domain $Dbase(d^i_j) \neq \emptyset$ of sub-actions, the set of (multi-dimensional) strategies S_i of a player i is given as:

$$S_i = \Pi_{d^i_j \in D_i} Dbase(d^i_j) \tag{1}$$

Decision-making problems $D = \bigcup_{i \in Q} D_i$ of all players make up factually *parameters of the internal model, cellModel*. Decision-making problems belonging to the subset $\{d \in D; |Dbase(d)| > 1\}$ are—from the modeling point of view—*unknowns*. Problems $\{d \in D; |Dbase(d)| = 1\}$ are for the sake of the better generality and modeling flexibility left as decision-making problems and denoted as *constants*. Player i having $|S_i| > 1$ is a *strategic player*. On the other hand, player i with $S_i = \{s^i_1\}$ is the *participating player with constant behavior* s^i_1. Configuration of variables and constants in a model is left to the experimenter, who works with the model and sets queries for the model by specifying $Dbase(d)$ for individual parameters $d \in D$.

2.3 Solving the game level

Let us have a game $\Gamma = (Q; S; U; C)$, where Q together with $(S_i)_{i \in Q}$ are considered to be the problem specification; $(U_i)_{i \in Q}$ is unknown in the beginning and is considered to be a interim result heading towards to equilibrium determination. We dispose of an application specific internal model *cellModel* able to enumerate $U(s)$ for all $s \in S$.

Let as call the equilibrium determination s^*_{CE} out of the specified game implementation based on Q, $(S_i)_{i \in Q}$ and *cellModel* to be a *mechanism*. The basic mechanism is shown in Algorithm 1. The computer procedure *cellModel* (s, C) is iteratively invoked for all profiles $s \in S$, so that we collect all $U(s)$. This basic mechanism is hard to compute as the set of profiles may be extremely large. Practically, we employ a sequence of clever heuristics minimizing the number of *cellModel* invocations to terminate the simulation in a reasonable time. This more efficient approach is out of the scope of this paper and can be seen in Hrubý (2008) and Hrubý and Čambala (2008).

Algorithm 1 Basic mechanism of solving the game level

for all $s \in S$ do:
 $U(s) := cellModel(s,C)$
$s^*_{CE} := CEsolver(Q,S,U)$

In the final step of the simulation, it is required to interpret the computed equilibrium point $s^*_{CE} = (\pi(s))_{s\in S}$ (the semantics of CE is given in Section 2.5). If there is a profile $s \in S$, such that $\pi(s) = 1$, then the game outcome is a unique equilibrium in the profile s. Otherwise, following the probability distribution s^*_{CE} we stochastically choose a single profile $s \in S$ that we be returned as the final result of the prediction.

2.4 Solving the level of internal model *cellModel*

Let *cellModel* (s,C) denotes an application specific model computing the hypothetical situation when the players $i \in Q$ in the game Γ play their actions s_i in the context of global constants $C = \{const_1, const_2, \ldots\}$. The internal model *cellModel* is expected to return $(U_i(s))_{i\in Q}$. This is a procedure that is invoked iteratively for all $s \in S$. The procedure itself may be of large time complexity t_{cm} depending on the particular application. The time complexity of the whole Algorithm 1 is then $|S| \cdot t_{cm} + ceComplexity(S)$, where $ceComplexity(S)$ is a complexity of $CEsolver$ algorithm. However, complexity of the equilibria computing is not studied here (see Papadimitriou 2005 for the general study of its complexity or Hrubý and Čambala 2008 for an advanced algorithm of computing the CE).

We would like to emphasize that the *cellModel* is not a simple $revenue - costs$ function. A practical example of its one particular implementation is shown in Section 3.3 as a part of the MCE model design. By denoting $U_i(s)$ we mean a particular and already known profit of the i-th player in $s \in S$. From the computer science point of view, $U_i(s)$ is a memory record. By denoting $cellModel(s)$ we mean an invocation of some computer procedure which takes some processor time to proceed.

2.5 Solving the game equilibrium (CE-Solver)

Correlated equilibrium is computable as a linear programming (LP) problem where we maximize the global objective function Z in (2) with probability variables $(\pi(s))_{s\in S}$ satisfying (3) and (4) to obtain the best solution for all players together. Pareto optimality is guarantied by (5). The LP problem is bounded by linear constraints in (5). Determination of correlated equilibria may be a large computational problem depending on the size of the game Γ. Its exact description is out of scope of this paper. Solution algorithms can be found in Papadimitriou (2005), Hrubý (2008) and Hrubý and Čambala (2008).

$$\max\ Z = \sum_{s\in S} \pi(s) Z(s) \tag{2}$$

$$\pi(s) \in \langle 0,1 \rangle \tag{3}$$

$$\sum_{s \in S} \pi(s) = 1 \tag{4}$$

$$\sum_{s_{-i} \in S_{-i}} \pi(s)\big(U_i(s_i, s_{-i}) - U_i(d_i, s_{-i})\big) \geq 0 \quad \forall i \in Q,\ \forall s_i, d_i \in S_i,\ d_i \neq s_i \tag{5}$$

$$Z(s) = \sum_{i=1}^{N} w_i U_i(s) \tag{6}$$

$Z(s)$ in (6) denotes one complex payoff of all players together in the strategy profile $s \in S$. These coefficients w_i are for everyone to tune for his own particular application. There is absolutely no general recommendation for that. Anyway, there are generally three approaches to that: $w_i = 1$, $w_i = 1/N$, w_i are different to each player (for example to normalize them if they are not similarly strong). We implement the first option, i.e. the simple summary of all payoffs $U_i(s)$. The behavior of players is bounded mostly by (5) and not by these w_i coefficients.

Solving the LP problem, we obtain an optimal point $s^*_{CE} = (\pi(s))_{s \in S}$, Z contains an optimal outcome for all players together. The constraints in (5) make the players not to deviate in this mixed profile. The vector s^*_{CE} is the unique wanted correlated equilibrium. See Aumann (1974) and Nau, Canovas and Hansen (2003) for the deeper mathematical description of CE. For the purpose of our modeling, this simplified explanation of the CE is fully sufficient.

In the practical simulation, the game is analyzed and reduced using algorithms described in Hrubý (2008). The relatively small reduced game is then put to this CE linear programming task. The implementation of the above algorithm following the method described in Hrubý (2008) is published as an independent tool called *CE-Solver* at CE-Solver (2008). The tool is based on a rather known library called GLPK (2008).

2.6 Overall view on the model design

To conclude this introductory section, we recall the main steps of the strategic model design:

(i) Collect all necessary information which is globally valid for the modeled situation—game context C.

(ii) Identify game players and collect their personal technical details regarding their production or consumption—set of players Q.

(iii) For all players $i \in Q$, describe their decision problems D_i and complete the game set of profiles S from players' particular sets of strategies S_i.

(iv) Design the internal model *cellModel* (s, C) able to compute consequences (payoffs) of players' actions $(s_i)_{i \in Q} \in S$.

The overall architecture of the prediction model arising from preparation steps (i–iv) and its layout is printed in Figure 1. In the next section, we will discuss deeply the design of the internal model, i.e. *cellModel*.

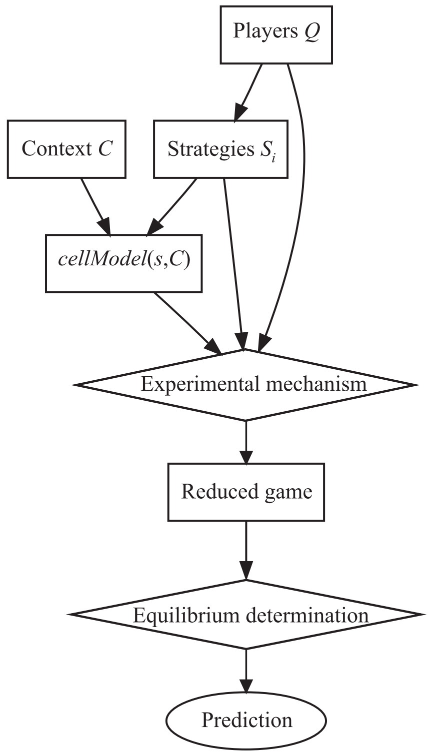

Figure 1. Overall architecture of the prediction model

The used methodology, which splits the model design to computing game utility functions and consequent determination of the equilibrium points, allows us to choose what equilibrium concept we want to implement in the model. We, as it was already mentioned, have chosen the correlated equilibrium defined by Aumann (1974). Obviously, correlated equilibrium has the same properties as mixed Nash equilibrium in the meaning that, in both sorts of equilibrium, no player can gain more by deviating from the equilibrium (mixed) profile. The main difference between Nash and Aumann's equilibrium must be seen in the way of computing them: Nash declared a state of payoff balance among the players, whereas Aumann defined what the players will never do—by sets of inequations over probabilities of profiles in (5). By solving the set of inequations (which is enormously easy), anyone can obtain a subspace of rational solutions in the game (geometrically a polytope). When having this subspace of reasonable solutions, the players search the right "synchronization device" to help them make their decisions. As we assume real players in the real world, we expect them to synchronize themselves using the current state of the situation: they know each other (their production abilities and planned consumption), they observed the last periods of the modeled situation, they know the context of the studied field and the current trends, and they all expect the whole set of players to maximize the economical profit. Moreover, these all facts are without any doubt a *common knowledge* in the situation and constitute the wanted *synchronization device*.

3. Model of Central Europe (MCE)

In this section we would like to present basic features of the model of electricity markets in Central Europe called Model of Central Europe (MCE). We include MCE to demonstrate and extend the theoretical methodology of Section 2. By using the MCE model, we analyze and predict the electricity trading in the given region, consisting of:

(i) A set of participating *national power networks*—national power networks are taken for nodes from the network point of view, since we assume there are no transport constraints between producers and consumers inside the network. The national power network is a market with its own production, trading and consumption. There are eight networks in MCE: E.ON (Germany), VE-T (Germany), PL (Poland), SK (Slovakia), CZ (Czech Republic), AT (Austria), HU (Hungary) and UA (Western Ukraine, which is from the network point of view connected to Central Europe).

(ii) A set of *interconnections* between individual networks with the set technical parameters—transit international power lines. These interconnections introduce constraints established by transmission system operators into the trading. Principles of power network operation also allow the trading between nodes without a direct interconnection, e.g. PL can supply AT transferring the supply through the neighbouring national networks. The topology of the system can be best seen in Figure 2.

(iii) A set of *producers*—a producer may supply his commodities to the national power networks respecting the international network constraints. For the reason of the extremely large context of the model, all national producers of a certain country all aggregated to a single producer representing that national power network. The producer within a certain country T thus aggregates all production units of all national electricity suppliers in T.

(iv) A set of *buyers*—a buyer is allowed make his purchases from producers according to network transport capacities. The buyer aggregates the total consumption in his network.

Players/producers are described by their production capacities; players/buyers by their domestic demand. These characteristics of players are considered generally available within the context of the MCE game (these data are *common knowledge* to all players). It is to some extent doubtful, whether it is correct to take these attributes for common knowledge among producers and buyers. Our research yields the conclusion that *producers are relatively well informed about each other*. Parameters of large

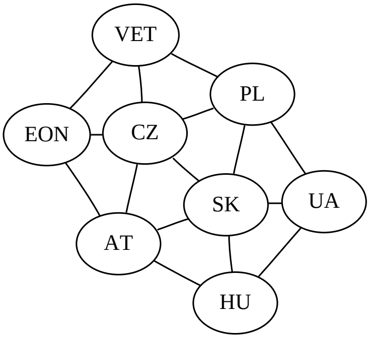

Figure 2. Topology of the interconnection network in MCE

power generation units are constant in the long term and these data are to some extent public and to some extent available for purchase (UCTE 2007). Moreover, power sector exhibits a long-term continuity with monotonically growing demand[2] and long-period operation of the generation sources. Those market participants that are too small in the meaning of the whole oligopoly situation cannot be distinguished at the MCE level; they are not able to affect the price and, anyway, the do not attempt to make any speculative deliberations.

Players/buyers make strategic decisions as well. Let us mention that any strategic player i with the strategy set S_i has to be able to compare for all $s_1^i, s_2^i \in S_i$ whether s_1^i is better than s_2^i or worse or equal. The comparison is possible only using player's utility function related to the strategies (cardinal utility). For that reason, we have to evaluate a financial benefit resulting from a given purchase contract.

MCE model contains a model of shares of individual consumer categories in the modeled countries $t \in T$—industry $w_n(t)$, service sector $w_s(t)$ and households $w_h(t)$, that $w_n(t) + w_s(t) + w_h(t) = 1$ holds $\forall t \in T$. For each category, the model evaluates the value added associated with the purchase and consequent consumption of 1 MWh.

3.1 Traded commodities within the MCE model

MCE introduces two categories of traded commodities: yearly base load/supply (YB, constant supply/load during all the 8760–8784 hours of the year) and monthly base load/supply (MB, constant supply/load in all hours of a particular month). There are in total 13 commodities, corresponding to one yearly commodity and 12 monthly commodities. We assume that each commodity has a different price at each national market. Power consumption, availability of sources and weather conditions (temperature, water, wind) exhibit considerable variations during the calendar year. This brings about considerable fluctuations in both nationally and internationally traded volumes and

[2] The situation with the general demand of electricity is currently unstable due to the fact of overall economic decrease, but that is not the issue of long-term forecasts where we assume almost predictable trends in production and consumption.

prices. Of significant importance will be the impact of emission allowances according to the EU regulation, which is also included in the MCE model. The effect of emission allowances on the price of electricity in EU is, however, an entirely separate phenomenon and is not studied in this paper.

The character of traded commodities (primarily of transport capacities) entails that year band contracts are traded first and monthly contracts are traded only after that. Let us recall that this situation leads to multi-dimensional decisions as the player makes his decision about YB with the outlook for the next 12 decisions about MB and looks for the optimum of all 13 commodities.

3.2 Decision-making problems of players in the MCE model

Let us define the MCE model more formally. The MCE model is a configuration of these four main attributes:

(i) Let $T = \{\text{CZ, SK, PL, AT, HU, VET, EON, UA}\}$ is a set of national power networks,

(ii) $P = \{p_k\}_{k \in T}$ is a set of producers and

(iii) $B = \{b_k\}_{k \in T}$ is a set of buyers. P together with B define the set of game strategic players $Q = P \cup B$.

(iv) Finally, function $Home : Q \to T$ assigns a domestic network to each player $i \in Q$.

Let us assume that each producer $i \in P$ disposes some available production capacity, described by a sequence of 12 values of monthly available power outputs $Mp_i = (m_1^i, \ldots, m_{12}^i)$. The minimum of this sequence, $YBp_i = \min(Mp_i)$, gives the available power output of the producer for the sale of commodity YB. In an analogous manner, we describe a buyer $i \in B$ by a sequence of his 12 monthly demands $Mb_i = (m_1^i, \ldots, m_{12}^i)$ and by his yearly constant demand $YBb_i = \min(Mb_i)$. When referring to a yearly band game, we mean such a game where producers want to sell their commodity up to the volume YBp_i and buyers wish to buy up to the volume YBb_i.

The sequence $(m_1^i - YBp_i, \ldots, m_{12}^i - YBp_i)$ gives the player's available production (demanded consumption) in particular months. Clearly, the player does not have to sell (buy) all his YBp_i (YBb_i). The producer (the buyer) has to make a decision regarding his level $0 \leq YB_i \leq YBp_i$ of the contracted amount in year base load; the rest $(m_1^i - YB_i, \ldots, m_{12}^i - YB_i)$ is left for the further month trading (the MB commodities).

The decision process in the YB part is fundamentally equal to the MB part. Respectively, the YB part contains twelve similar sub-games for twelve MB contracts of the same structure as the YB contracts. In the MB decision making, the player again decides his portion to sell home, to export, to keep in reserve. To present the methodology, we will concentrate on the strategically most important commodity, which is the yearly base load (YB). The Figure 3 shows a typical decomposition of the production available capacity (or scheduled consumption, i.e. the demand) to its YB part and twelve MB parts.

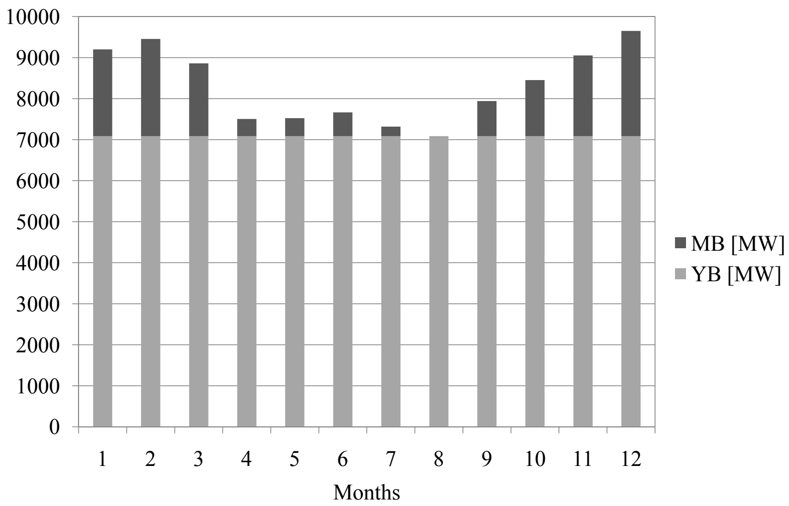

Figure 3. Decomposition of production or demand to its YB and MB parts

3.2.1 Decision-making problems of producers

Within the framework of the *yearly base* strategic commodity, producer i shall adopt the *multi-dimensional* strategic decision about (having YBp_i [MW] of the available production capacity):

(i) **Base price** C^i_{YB} [€/MW] of the yearly band.[3] The producer will regard this price as the minimum price at which he could sell the yearly band at any market. This price allows to estimate his anticipated financial profit per 1 MWh of YB, which he requires. If we strictly assume one producer per power network k in the basic variant, then we can define C^k_{home} as the prevailing price in the network concerned. Formally, $C^k_{home} := C^p_{YB}$, $p \in P$ such that $Home(p) = k$.

(ii) **Volume** Oh^i_{YB} [MW] offered at the domestic market $Home(i)$ for price C^i_{YB}.

(iii) Neighboring markets to which the producer i will export his production. This decision can be broken down to three subdecisions. First, the producer must decide about the **total volume** Oe^i_{YB} **[MW] of production for export**. Second, he decides about the volume of the commodity he wants to offer at each particular foreign power market (i.e. national power network). Third, he decides about a bid for the auction for transport capacities.

[3] We should distinguish between a payment for a produced/consumed 1 MWh and contracted price for 1 MW of the yearly continuous supply of the commodity. The cost of a 1MW of the YB commodity may be for example €60 and the buyer pays 60 times number of hours in the year (8760 or 8784), i.e. 525,600 or 527,040 EUR.

Hence, producers $i \in P$ with $Home(i) \neq k$ where the price $C^k_{home} > C^i_{YB}$ will deliver their commodity to networks $k \in T$ at price C^k_{home}−€1. This is due to the fact that a player participates in an auction for transport profile only after the contract with buyer b with $Home(b) = k$ is negotiated. Therefore, we do not assume, that the price requested by the producer should intentionally be significantly different from price C^k_{home}.

(iv) Offering his capacity as a positive spinning reserve Or^i_{YB} [MW]. The spinning reserve market is not studied in this paper. The amount of Or^i_{YB} is included only for the production balancing reasons.

(v) **Reservation** of a part of the production Om^i_{YB} [MW] for monthly commodity markets. The producer decides about to sell his YB production capacity as YB commodity or to split that to 12 MB contracts for probably better price.

(vi) Canceling the production On^i_{YB} [MW] altogether and acquisition of the associated profit from cancelled production instead (sale of emission allowances). Such a situation is strategically complex and it is not studied here.

Apparently, a producer i wishes to sell all his yearly available production capacity YBp_i, hence

$$YBp_i = Oh^i_{YB} + Oe^i_{YB} + Or^i_{YB} + Om^i_{YB} + On^i_{YB}$$

Each producer i must, therefore, make his own decision about the breakdown of his total yearly available production capacity YBp_i into the components described above, including their prices. Finally, the multi-dimensional pure strategy of producers is a vector:

$$s^i_p = (C^i_{YB}, Oh^i_{YB}, Oe^i_{YB}, Om^i_{YB}) \tag{7}$$

For the purpose of simplicity, we omit its parts Or^i_{YB} and On^i_{YB}. To specify the domains of its internal parts, $C^i_{YB} = \langle 0, priceMax\rangle$ (where $priceMax$ is the maximum reasonable price in this sort of industry), $Oh^i_{YB}, Oe^i_{YB}, Om^i_{YB} \in \langle 0, YBp_i\rangle$ are amount of production expressed in MW (8) or in percentage of YBp_i (9) which is preferred in the following case study. We would like to emphasize, that the real price of a commodity is always bounded somehow, hence the model may work in the discretized and final set of profiles.

$$Oh^i_{YB} + Oe^i_{YB} + Om^i_{YB} = YBp_i \quad [MW] \tag{8}$$

$$Oh^i_{YB} + Oe^i_{YB} + Om^i_{YB} = 100\% \quad [YBp_i] \tag{9}$$

3.2.2 Decision-making problems of buyers

Buyer i specifies his elasticity curve and his plan of purchases so as to meet his demand and to minimize his spending. He makes a strategic decision as to what part he will buy as the yearly band contract and what part he will buy in 12 separate monthly contracts. The month-specific demand is dealt with in the relevant months.

The yearly band YB is the strategically most interesting item again, where the buyer i must decide about (having his YBb_i demand):

(i) **Reference price** C^i_{ref} [€/MW] of electricity in his home network. This is the price at which he would like to buy all his demand YBb_i. As an example, we may take the prevailing price from the previous year (prices in the discussed market grow monotonically in the last 10 years).

(ii) **Maximum price** C^i_{max} [€/MW] in his network. The buyer i will certainly not buy any volume at prices exceeding C^i_{max}. Energy markets are to some extent under the supervision of national regulatory authorities and the price C^i_{max} can be best understood as that price when the regulatory authority could intervene in an otherwise free competition.

(iii) **Volume** Ob^i_{YB} [MW], which he would like buy within the YB framework, $0 \leq Ob^i_{YB} \leq YBb_i$.

(iv) Reservation of a part of the consumption Om^i_{YB} [MW] for monthly commodity markets. The buyer may assume that he would done better when purchasing his YB demand as 12 MB contracts (e.g. each supplied by a different producer).

(v) Volume of positive spinning reserve Or^i_{YB} [MW] which he wants to buy. Again, the spinning reserve market is not under study here.

(vi) **Elasticity coefficient** δ_i [MW/€] expressing the decreasing interest of the buyer in the commodity with increasing commodity price. See Figure 5 to have an example of buyer's response to the demanded price.

Finally, the multi-dimensional pure strategy of buyers is a vector (10). We understand Om^i_{YB} to be a complement to YBb_i. Let us express the volumes (11) and (12) similarly as in the producer's case.

$$s^i_b = (C^i_{max}, C^i_{ref}, \delta_i, Ob^i_{YB}) \tag{10}$$

$$Ob^i_{YB} + Om^i_{YB} = YBb_i \tag{11}$$

$$Ob^i_{YB} + Om^i_{YB} = 100\% \quad [YBb_i] \tag{12}$$

The pure game-theorists might argue against the concept of base price, reference price and maximum price saying that these prices should be a result of the game-theoretical modeling and reasoning, and not its parameters. We, however, look for some equilibrium point of the overall agreement among all the players (correlated equilibrium in MCE case) in some particular finite set of profiles where the prices are its important part.

3.3 Design of the internal model $cellModel_{YB}$

As we have already mentioned, the *cellModel* is a procedure (also referred as an "oracle") computing the utility $U(s)$ of all players when playing the profile $s \in S$ and a global context C. Mathematically, *cellModel* is a function

$$cellModel\ (s, C) : S \times C \to \mathbb{R}^N$$

From the modeling point of view, *cellModel* models what all would happen if the players $i \in Q$ would play s_i in context of C in the reality. The context C is the global state of the system unchangeable during the game (e.g. state of the transmission capacities, legislation). Let us remind that the strategies s_i are multi-dimensional, of the form (7) for the producer and (10) for the buyers. Moreover, we would like to remind that we assume the all contents of the $cellModel_{YB}$ including all information to be a *common knowledge* to all players.

At the top level, the $cellModel_{YB}\ (s,C)$ is a sequence of these following main actions (phases):

(i) **Offer/Prepare**. Players/producers make their bids to the markets regarding their strategy s_i, i.e. they offer Oh^i_{YB}, Oe^i_{YB}, Om^i_{YB}. The players/buyers announce their demand regarding their strategy s_i, i.e. Ob^i_{YB} and their reference and maximum price, i.e. C^i_{ref}, C^i_{max}.

(ii) **Trading**. The markets (players/buyers) selects some bids to accept. They verify the contracts.

(iii) **Production**. The players/producers receive an information about the accepted bids and optimize their production plants to produce the contracted YB volume of electricity.

(iv) **MB contracts**. The players (producers and buyers) play twelve similar games, nested to this YB game, to contract the twelve MB commodities.

(v) **Conclusion**. The players enumerate their final financial profit made in the situation of the profile s. The profit also includes the profit made in MB nested games (or other business done—sold emission allowances, spinning reserve, etc.).

Now, let us describe the actions in details. Let a bid is a structure $Bid = (t_f, t_t, a_o, a_v, a_s, price, tax)$, where t_f is a network of origin, t_t network of destination and $t_f, t_t \in T$; a_o is an amount offered, a_v amount verified, a_s amount sold. The *price* denotes the demanded cost of the commodity and *tax* the fee which the player agrees to pay for the unit of transmission capacity (international power lines). The fee *tax* is de facto the player's bid in the FB auction. Clearly, $tax = 0$ if $t_f = t_t$ (player is selling to his home network) and $tax > 0$ should hold otherwise.[4]

The following sections assumes that all mentioned constants or variables are expressed in the context of a particular strategic profile $s \in S$ when $cellModel\ (s)$ is invoked.

3.3.1 Offer/Prepare phase

The buyer i will announce his will to buy up to $demand_{Home(i)} = Ob^i_{YB}$ [MW] in the framework of the YB commodity.

[4] Charging for the transmission is great problem new in discussion between producers and Transmission System Operators (TSO): producers claim that *tax* is supposed to have an regulatory purpose and TSOs should not make profit in the FB auction. Anyway, we strictly assume *tax* to be non-zero.

The producer i puts his bid

$$hbid_i = (Home(i), Home(i), Oh^i_{YB}, Oh^i_{YB}, 0, C^i_{YB}, 0)$$

to his domestic market. We assume that the major portion of the consumption in each country is covered by the domestic producer. The producer sets $C^{Home(i)}_{home} := C^i_{YB}$.

Let $K_i \subset T$ is a set of countries, where $\forall k \in K_i : C^k_{home} > C^i_{YB}$. The producer i wants to export to these countries (for $C^k_{home} - 1$ price). To complete this exporting subdecision, the producer has to decide the break-down of Oe^i_{YB} amount of the commodity to these countries, as we assume that $\sum_{k \in K_i} demand_k \gg Oe^i_{YB}$ and the network constraints will not allow him to export all his Oe^i_{YB} to a single country $k \in K_i$ (e.g. with the highest price C^k_{home} or shortage of supply $Oh^j_{YB} \ll demand_k$, where $j \in P$, $Home(j) = k$). Setting the auction bids (the price of the transportation capacity) is the second part of that sub-decision.

The behavior of producers in the flow-based multi-object auction is a topic for another journal paper. Mostly, the players are risk-averse (Krishna 2002) in the long-period contracts (they made the supply contracts and now they seriously need to obtain a transmission permission to deliver the supply), hence they bid $tax = C^k_{home} - C^i_{YB} - 1$ as the maximum they can afford to pay.

Finally, the producers put their exporting bids

$$ebids_i = \{(Home(i), k, a^k_o, 0, 0, C^k_{home} - 1, C^k_{home} - C^i_{YB} - 1) | k \in K_i\},$$

such that for the particular shares of the exporting amounts holds

$$\sum_{k \in K_i} a^k_o = Oe^i_{YB}.$$

Let

$$bids_i = \{hbid_i\} \cup ebids_i$$

are the total bids of the producer-player i sent to the markets $\{Home(i)\} \cup K_i$. The set of overall bids is then:

$$Bids = \bigcup_{i \in P} bids_i$$

3.3.2 Auction for the international transmission capacity (Flow-Based Method)

As we already mentioned, the international trade is constrained with a limited transmission capacity over the national networks. To select and regulate those producers allowed to transmit, various sorts of auctions are organized by TSOs. As the electricity business is getting more and more international, the auctions are getting centralized and covering a larger geographical area (e.g. Central Europe, North Europe–Nordpool, etc.). FB auction legislation is currently in development and possibly being implemented, no matter the protests of various producers in involved countries.

The MCE model implements the Flow-Based Method (FB auction) in the following manner: Those wishing to use cross-border capacities are allocated available capacity using an algorithm that brings maximum benefit to the consortium of operators. For practical purposes, this means that the impact of each planned trading event is evaluated on the basis of pre-calculated distribution coefficients (so-called PTDF coefficients, Power Transfer Distribution Factor, see Vukasovic and Skuletic 2007). After that, a combination of mutual transactions is selected in such a manner that it brings maximum profit to the operator (here, "operator" denotes Central European power systems as a whole) and at the same time considers capacity constraints of cross-border profiles. For this purpose, linear optimization is used.

Let $Bids_{FB} = \{b \in Bids | t_f \neq t_t\}$ are the bids intended for export and hence they must be selected by the FB auction. Because of physical laws of electricity flows in circuits, each contract $b \in Bids_{FB}$ supplying a_o MW from t_f to t_t is physically spread over the whole network (see Figure 2). The physical electricity flows (including their orientation) are demonstrated on an example of transferring 100 MW from CZ to E.ON (see Table 1). Positive number in a cell (from/to) gives the number of megawatts flowing in this direction (the negative number indicates a reverse flow and thus an increase of the available capacity in the reverse direction). The complete state of the transmission system is computed as a superposition of all individual contracts. However, $\sum_{Bids_{FB}} a_o$ would probably exceed the technical capacity of the transmission system and thus it is required to allow only some contracting bids to be realized.

Table 1. Example of the PTDF coefficient for from CZ to E.ON transmission

from/to	CZ	SK	PL	AT	HU	VE-T	E.ON	UA	Outside
CZ	−100.0	9.2	14.5	16.6	0	31.9	27.8	0	0
SK	−9.0	0	1.8	0	6.2	0	0	1.0	0
PL	−14.2	−1.8	0	0	0	16.0	0	0	0
AT	−16.1	0	0	0	0.4	0	14.1	0	1.7
HU	0	−6.1	0	−0.4	0	0	0	0	6.5
VE-T	−30.9	0	−16.0	0	0	0	47.0	0	0
E.ON	−26.9	0	0	−13.9	0	−46.2	100.0	0	−13.0
UA	0	−1.0	0	0	0	0	0	0	1.0
Outside	0	0	0	−2.0	−7.6	0	10.8	−1.2	0

Let $Links \subseteq T \times T$ is a set of real existing interconnection lines between the countries. (*Links* is a symmetric relation. See Figure 2 for its content.) Each line $(f,t) \in Links$ is given some decided capacity, i.e. there exist a function $Cap(Links) \rightarrow \mathbb{R}$ assigning an available capacity $[MW]$ to each line. The important fact is that $Cap(f,t)$ is generally different to $Cap(t, f)$ due to the technical aspects of the whole network. The function *Cap* is set as an agreement among all cooperating national transmission system operators. *Cap* is a common knowledge to all players. By using a certain capacity c of a line (f,t), we increase the available capacity of (t, f) with extra c.

In the FB auction, the total revenue (in each hour 1 ... 8760–8784 of the year) of the auctioneer is maximized:

$$\max\ Z = \sum_{b \in Bids_{FB}} a_v^b \cdot tax \tag{13}$$

The optimization process called the Flow-base method is computed as a linear programming task with an objective function (13), with LP variables $\{a_v^b\}_{b \in Bids_{FB}}$ (a_v^b is a variable associated with the bid $b \in Bids_{FB}$):

$$\forall b \in Bids_{FB} : a_v^b \in \langle 0, a_o^b \rangle$$

and the constraints:

$$\forall (f,t) \in Links : \sum_{b \in Bids_{FB}} a_v^b \cdot PTDF_{t_f,t_t}(f,t) \leq Cap(f,t)$$

$PTDF_{f,t}$ is a PTDF-matrix modeling the flows from $f \in T$ to $t \in T$ through the Central European network. It also means that $PTDF_{f,t}$ is required for all $(f,t) \in T \times T$.

The flow-based auction mechanism terminates with variables $\{a_v^b\}_{b \in Bids_{FB}}$ containing the amount of the commodity allowed to be exported from t_f network to t_t network. The revenue of the auctioneer is maximized in (13). Currently, there is a discussion between the government institutions responsible for the electricity network and the traders whether the criteria in (13) are fair or not. The alternative objective function shown in (14) maximizes the overall trade among the national networks (this, however, would not motivate the players to bid their true value). Studying the possible auction mechanisms for this problem would be a topic for another paper.

$$Z = \sum_{b \in Bids_{FB}} a_v^b \tag{14}$$

The overall flow of bids processing is displayed in Figure 4.

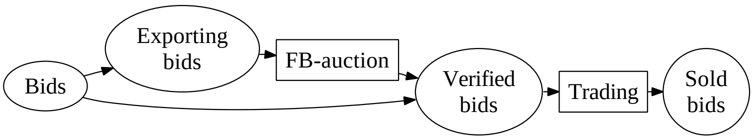

Figure 4. Flow of bids during the Prepare, FB auction and Trading phase

3.3.3 Trading phase

Let $Offers_i = \{b \in Bids | Home(i) = t_t \wedge a_v > 0\}$ is a sorted list of bids received by the buyer $i \in B$. The buyer sorts them ascendantly by *price* and buys up to his demand Ob_{YB}^i (see Figure 5).

At the end, *Bids* are transformed to a list of bids where $a_s \in \langle 0, a_v \rangle$ shows the sold amount within the offered bids. Thus, $a_s \leq a_v \leq a_o$ holds for all $b \in Bids$. The decrease $a_v \leq a_o$ is caused by the FB auction, the decrease $a_s \leq a_v$ by the buyer.

Let us remind that producers export to countries $k \in T$ for $C_{home}^k - 1$ price to ensure that their bids will be accepted by buyers.

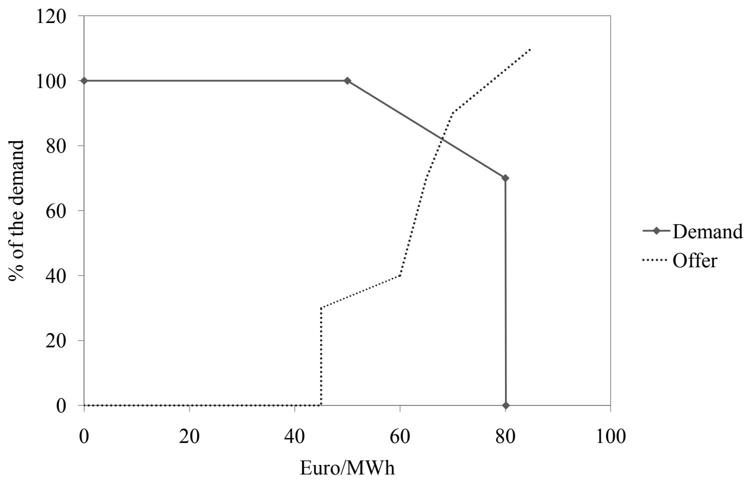

Figure 5. Elastic demand with $C^i_{ref} = 50$ [€/MW] and $C^i_{max} = 80$ [€/MW] together with the supply curve (sorted list of bids)

3.3.4 Production phase

Let

$$Contracts_i = \{b \in Bids | Home(i) = t_f \wedge a_s > 0\} \tag{15}$$

is a list of accepted contracts of the producer $i \in P$. The producer p has to arrange his production scheme to fulfill the contracted amount in (16) in the YB commodity supply.

The producer i dispose of a set of production sources (power plants) $R_i = \{r^i_1, r^i_2, \ldots\}$. We define these following attributes for the set R_i:

(i) Installed capacity $Icap(R_i) \rightarrow \mathbb{R}$ [MW]

(ii) Available (disponible) capacity $Acap(R_i, M) \rightarrow \mathbb{R}$ [MW] fluctuating during the months $M = \{1, \ldots, 12\}$

(iii) Production cost $Prodc(R_i) \rightarrow \mathbb{R}$ [€/MWh]

(iv) Fixed cost $Fixc(R_i) \rightarrow \mathbb{R}$ [€/MWh]

$Icap$, $Prodc$ and $Fixc$ remain constant during the year. Just $Acap$ fluctuates during the months, mostly for scheduled repairs and season reasons. The constants yh and $mh(M)$ denotes the length of a year in hours (8760–8784 hours) and the length of months $m \in M$ in hours, $M = \{1, \ldots, 12\}$.

$$Sold_i = \sum_{b \in Contracts_i} a_s \tag{16}$$

$$FixedCosts_i = yh \cdot \sum_{r \in R_i} Fixc(r) \cdot Icap(r) \tag{17}$$

$$Revenues_i = yh \cdot \sum_{b \in Contracts_i} (price - tax) \cdot a_s \tag{18}$$

$$ProductionCosts_i = \sum_{r \in R_i} \sum_{m \in M} v_m^r \cdot Prodc(r) \cdot mh(m) \tag{19}$$

$$ProfitProducer_i = Revenues_i - FixedCosts_i - ProductionCosts_i \tag{20}$$

The producer i has to pay his fixed costs (17) during the year. His production cost (19) is based on his contracted sells (16) with the outcome (18). The production cost (19) is minimized in another LP task with variables (21) and constraints (22). Other constraints regarding the production (emission limits, fuel limits, special characteristics of particular sources, etc.) can be added as well.

$$\forall r \in R_i, \forall m \in M : v_m^r \in \langle 0, Acap(r,m) \rangle \tag{21}$$

$$\forall m \in M : \sum_{r \in R_i} v_m^r = Sold_i \tag{22}$$

The buyer's payoff is made by his added value (revenues) minus costs of the purchase. Let us consider the following equations (23)–(27). The coefficients AV_n, AV_h, AV_s denote the value added [€/MWh] with every 1MWh consumed in the industry, households and services. Their particular values are set by an expert operating the model.

$$BContracts_i = \{b \in Bids | Home(i) = t_t \wedge a_s > 0\} \tag{23}$$

$$Purchased_i = \sum_{b \in BContracts_i} a_s \tag{24}$$

$$CostPurchase_i = yh \cdot \sum_{b \in BContracts_i} a_s \cdot price \tag{25}$$

$$AddedValue_i = yh \cdot Purchased_i \cdot (w_n(i)AV_n + w_h(i)AV_h + w_s(i)AV_s) \tag{26}$$

$$ProfitBuyer_i = AddedValue_i - CostPurchase_i \tag{27}$$

3.3.5 MB phase

The game for YB includes 12 nested games on trading with MB. Players/producers in each month z offer volumes $m_z^i + Om_{YB}^i + Rest_{YB}^i$, where $Rest_{YB}^i = Oh_{YB}^i + Oe_{YB}^i - Sold_i$ is the unsold part of their yearly production band.

In an analogous manner, the same applies for buyers exhibiting a demand. The game for MB corresponds basically to that for YB with the difference that buyers are assumed to have a stronger interest to buy than in case of YB (he does not apply his elasticity coefficient δ_i in MB).

In the MB phase, the computing process described above is repeated twelve times to compute the players domestic contracts, export etc. There is no fundamental difference to $cellModel_{YB}$ processing, except the demand curve which is flat—the buyers purchase almost for any price.

$$profitMB = \sum_{m\in\{1,\dots,12\}} cellModel_{MB}(s, C\cup\{m\})$$

3.3.6 Conclusion phase

Players/producers compute their financial profit achieved in YB trading together with twelve MB contracts (and possibly others, if implemented). Buyers compute their financial profit from the realized purchase.

The internal model *cellModel* (s,C) has been semi-formally described. When invoked, it passes the phases 1–5 and terminates with profits $U(s) = (U_i(s))_{i\in Q}$ where:

$$U_i(s) = \begin{cases} ProfitProducer_i + profitMB_i & i\in P \\ ProfitBuyer_i + profitMB_i & i\in B \end{cases} \tag{28}$$

3.4 Simulation run

The MCE model, or any similar to that, consists of its main model part (players, strategies, game rules) and its internal model part *cellModel* (see Section 3.3). We briefly described both modeling parts. They are put together in the Algorithm 1.

In the simulation, the players make their decisions from their strategy sets S_i through the game analysis of utilities $U_i(s)$ computed by *cellModel* for each $s\in S$.

4. Implementation of the simulation experiment (a case study)

In this chapter we would like to present the region modeled (see Figure 6), market players, implementation procedure and results of the basic experiment carried out with the model for the year 2009. In this way we predict the future state of the electric power system in the Central European region (with the main emphasis on the Czech power system) in the horizon of 1–10 years.

We start with our own estimates of power demand, power generation availability and conditions of the power network, which we compare with the estimates from other sources (e.g. UCTE System Adequacy Forecast 2007). As input data are just estimates and the calculation is just a model, the results my depart from the real situation.

4.1 Players in the MCE model

Each country is represented by two players; the first player is the aggregate power producer for the given country, the second player is the aggregate buyer. German power system is an exception; for simulation purposes, this system is divided in two separate regions with working names VE-T and E.ON (see Figure 6). This figure also shows transmission capacities for winter 2008–2009.

The model thus includes 16 players, each with his own individual strategy of behavior. The simulation experiment is expected to result in the determination of the strate-

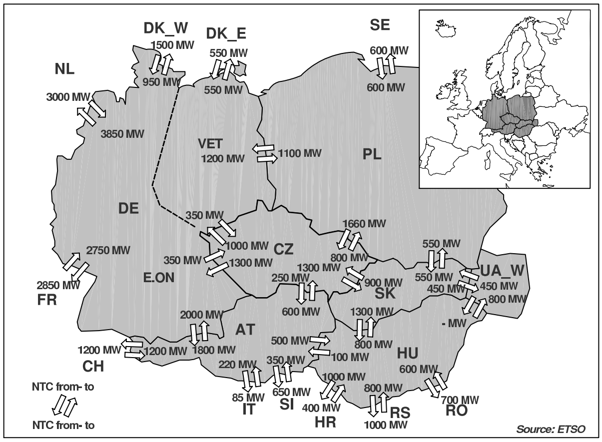

Figure 6. Model of Central European region

gic behavior of power producers and buyers in the yearly (YB) and monthly (MB) base load commodities.

4.1.1 Power producers and installed power generation capacities in individual power systems

The game entirely excludes power sources whose offers are for legislative reasons always accepted (hydro, wind and photovoltaics). Their production will be subtracted both on the production and purchase side. Power sources taking part in the competition (fossil and nuclear fuel, biomass) are described by a number of parameters. They include net installed capacity, consumption of auxiliaries, failure rate, planned outages, type and price of fuel, specific fuel consumption, fixed costs and type of operation (must-run, non-constrained, standing reserves). Each player's portfolio may contain separately modeled power units (each model unit corresponds to a real generating unit), units modeled by groups (each model unit corresponds to one aggregate power plant) and virtual units (each model unit aggregates several power plants with total power output lower than 50 MW).

An example of a summary table of power sources owned by individual players participating in the model is given in Table 2. It is not possible to display the whole database of the sources, we provide just a summarization.

4.1.2 Buyers and consumption

In accordance with the concept of buyers as players, each power system is on the buyer's side represented by a single buyer only. The reference value of the demand for electricity, which each player wishes to satisfy, is determined from the predicted

Table 2. Power sources by players in 2009

Power system	Installed capacity [MW]		Number of sources		
	Gross	Net	Individual	Group	Virtual
CZ	15312	14093	79	24	7
SK	3656	3399	28	–	1
PL	34534	31039	297	–	25
AT	5896	5605	57	–	–
HU	8300	7846	95	–	2
VE-T	15363	14051	82	18	5
E.ON	79196	72553	285	39	5

evolution of macroeconomics data for all economic sectors. The basic break-down of the power demand estimate for the year 2009 is given in Table 3. The complete demand for YB and MB commodities is given in Table 4.

We should emphasize that we do not model the demand itself. The demand is an input for our forecasts and comes from other models.

Table 3. Estimated consumption of electricity in 2009 [TWh]

Power system	Industry and services	Households	Power losses	Total
CZ	46.7	15.2	5.4	67.3
SK	21.1	6.8	2.4	30.2
PL	96.1	24.0	15.1	135.2
AT	48.5	16.9	3.6	69.0
HU	29.1	11.7	3.8	44.5
VE-T	50.0	17.8	3.5	71.2
E.ON	355.0	126.4	24.9	506.4

Table 4. Demand for yearly and monthly bands [MW]

Commodity	CZ	SK	PL	AT	HU	VE-T	E.ON
YB	7086	2545	16351	4376	5109	9044	60752
MB 1	2116	1448	4718	3850	801	162	4673
MB 2	2370	977	2835	3058	443	771	4338
MB 3	1774	659	2046	2088	308	113	610
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
MB 12	2564	1308	3777	3611	415	33	3944

4.2 Strategies of players

An important feature of each player-producer in simulation calculations is his decision-making strategy regarding the price and volume of power offered at domestic markets, power offered for export and the offer of power reserves. In case of power purchasing players (buyers), the main components of the strategy are the volume of purchased electricity, a proper choice of the reference price of electricity and an optimally set elasticity of power demand. The components d_j^i (see Section 2.2) of the strategy may be entered as a fixed value (e.g. =100) or as an interval with a suitable step (e.g. 50:70:2 in form $min\!:\!max\!:\!step$ which gives a set $min, min + step, \ldots, min + x \cdot step \leq max$. The complete strategy set of the player i is then generated as (1). Examples of strategy configuration files both for producers and buyers are given in the Tables 5 and 6.

Table 5. Producer strategy configuration file

Power system	C^i_{YB} [€/MW]	% of YBp_i		
		Oh^i_{YB}	Oe^i_{YB}	Om^i_{YB}
CZ	50:70:2	80:100:5	0:20:5	0:10:5
SK	55:75:2	100	0	0
PL	45:65:2	80:100:5	0:20:5	0:10:5
AT	55:75:2	100	0	0
HU	60:80:2	100	0	0
VE-T	50:70:2	80:100:2	0:10:2	0:10:2
E.ON	50:70:2	80:100:2	0:10:2	0:10:2
UA	30:50:5	0:50:10	50:100:10	0

Table 6. Buyer strategy configuration file

Power system	C^i_{max}	C^i_{ref}	δ^i	Ob^i_{YB}
	[€/MW]	[€/MW]	[MW/€]	[%]
CZ	80	53	170	100
SK	85	61	45	100
PL	70	50	550	100
AT	85	59	200	100
HU	90	68	150	100
VE-T	85	59	180	100
E.ON	85	59	2000	100
UA	45	40	100	100

To keep this case study rather simple, we let the buyers to be only participating players with constant behavior (i.e. their $|S_i| = 1$). Let us note that some of the players

(SK, AT, HU) will not export, thus we do not enter the exporting strategies for them (they set $Oh^i_{YB} = 100\%\ YBp_i$).

The configuration of strategies of the players gives the final set of profiles with total size approximately $1.2 \cdot 10^{15}$ of strategy profiles. Sizes of elementary strategy sets are shown in Table 7.

Table 7. Number of strategies of the players

Role/network	CZ	SK	PL	AT	HU	VE-T	E.ON	UA
Producer	165	11	165	11	11	561	561	105
Buyer	1	1	1	1	1	1	1	1

4.3 Results attained in simulations

All inputs were entered and the MCE model terminates in an equilibrium point giving the following results. The experiment was computed using a computer with 8 x CPU Intel Xeon 2.66 GHz and 16 GB RAM. It took approximately 17 minutes to obtain the simulation result (see Hrubý 2008, for a technical description of the algorithms solving game reduction and CE determination).

Technically, the simulation of game $\Gamma = (Q = P \cup B; (S_i)_{i \in Q}; (U_i)_{i \in Q}; C)$, where $U_i(s) = \mathit{cellModel}\ (s, C)\ \forall s \in S$ and game context C, reduces the game Γ to its strategic equivalent $\Gamma^r = (Q; (S^r_i)_{i \in Q}; (U^r_i)_{i \in Q}; C)$ (Γ^r is best-response equivalent to Γ) where $|S^r| \ll |S|$—see Hrubý (2008) for more detail on its algorithmic implementation (FDDS reduction method). Finally, an equilibrium point s^*_{CE} predicting the players' probable behavior in form of correlated equilibrium is determined using CE-Solver algorithm, also in Hrubý (2008).

Let us remind that $s^*_{CE} = (s^*_i)_{i \in Q}$ contains the decisions of players $i \in Q$ (see (7) and (10) to get their data structure). Statistics presented in the resulting tables are the computational outputs of *cellModel*.

4.3.1 Results of simulations of yearly band trading

Final prices of electricity in the yearly band in individual power systems, resulting from simulation calculations, are shown in Table 9. They agree relatively well with the results of power trading at power exchanges for the year 2009, which are already available. Meeting the demand (Table 4) commercially is presented in Table 8. The table clearly shows the impact of demand elasticity on commercial supplies needed to meet yearly band. The volume of the purchased yearly band is always lower than the volume demanded at the reference price (see Table 8). At the same time, the table clearly shows how the electricity not purchased within the yearly band framework is spread to individual months.

Table 8. Purchased volume in power systems [MW] for yearly and monthly bands

Commodity	CZ	SK	PL	AT	HU	VE-T	E.ON
YB	6576	2056	16351	3176	4659	8684	51778
MB 1	2626	1111	4718	2107	1251	522	13647
MB 2	2880	856	2835	2002	893	1131	13312
MB 3	2284	835	2046	2274	758	473	9584
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
MB 12	3074	1340	3777	3305	865	393	12918

Table 9. Final prices of electricity in yearly and monthly bands [€/MW]

Commodity	CZ	SK	PL	AT	HU	VE-T	E.ON
YB	56	65	53	65	71	61	63
MB 1	63	67	64	75	73	53	55
MB 2	64	75	52	73	70	59	56
MB 3	57	71	46	68	71	57	57
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
MB 12	59	62	57	72	70	53	56

4.3.2 Results of simulations of monthly band trading

A brief overview of monthly results is shown in Tables 8 and 9. In the course of simulation, monthly demand has been increased by that part of the demand that has not been met in yearly contracts. Demand elasticity is no more considered in monthly trading. Demanded electricity is in most months traded in full. However, the buyers do not always contract their whole demand. Failure to cover monthly demand in full in some power systems (primarily Slovakia and Austria) is due to the lack of internal ability of these power systems to cover domestic load by supplies from domestic power sources. This failure also reflects drawbacks of the Flow-Based Method, because its application means that the available capacity of certain cross-border profiles is quickly exhausted and thus other needed trades are blocked.

Other simulation outputs, e.g. business and physical flows, fuel consumption, scheduling of power sources, balances in individual power systems, traded volumes and prices of emission allowances, business results of power producers etc., are deducted from the game equilibrium. These secondary outputs are useful for a wide spectrum of analyzes of any possible kind.

5. Conclusion

We presented a methodology for the modeling of electricity markets using the tools of the mathematical game theory. We described the complete process of prediction model development from initial specifications up to the final result interpretation. The methodology makes a very general framework applicable easily in similar modeling projects. From the algorithmic game-theoretical point of view, the paper develops and extends two rather new concepts.

Two-level architecture of a strategic decision model. This approach allows the decomposition of the whole problem to a pair of a relatively general experimental mechanism and an application specific sub-model, called the internal model (*cellModel*) here. The experimental mechanism defines the way of computing the complete strategic state space (set of profiles) as well as the way of analyzing it with the aim to find its equilibrium point. This method is well suitable for the computer processing. We can find it in MCE model as a basic concept. Technical (algorithmic) approaches to implement such an experimental mechanism were published in Hrubý (2008) and recalled here.

Modeling of *structured (multi-dimensional) decisions*. Real-life decision situations are full of decision alternatives and their transformation to a computer model might be rather difficult. Presented methodology simplifies their efficient implementation in a computer model. Structured decision making was demonstrated in MCE, where the player-producer thinks about breaking down his production capacity to various commodities and markets.

The main core of the paper is concentrated to a rather detail description of our MCE model, which consists of the game design, its strategy state space, internal model and the equilibria concept. This part is considered to be the main contribution of the paper.

Finally, the functionality of the model was shown on an example of the analysis for the year 2009. This example included realistic input data on power demand, availability of power sources and conditions of the international transmission network.

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